# Τυπολόγιο Κυματοδηγών

## ΛΥΣΕΙΣ ΤΗΣ ΕΞΙΣΩΣΗΣ HELMHOLTZ

### Α) ΣΕ ΚΑΡΤΕΣΙΑΝΕΣ ΣΥΝΤΕΤΑΓΜΕΝΕΣ

$$\nabla_{xy}^2 \Psi + k_C^2 \Psi = 0, \quad k_c^2 = \omega^2 \varepsilon \mu - \beta^2$$

$$\Psi(x, y) = [A_1 \cos(k_x x) + B_1 \sin(k_x x)] \cdot [A_2 \cos(k_y y) + B_2 \sin(k_y y)], \quad k_c^2 = k_x^2 + k_y^2 > 0$$

### Β) ΚΥΛΙΝΔΡΙΚΕΣ ΣΥΝΤΕΤΑΓΜΕΝΕΣ

$$\left(\nabla_{\rho,\varphi}^2 + k_c^2\right)\Theta(\rho,\varphi) = 0, \quad \nabla_{\rho,\varphi}^2 = \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial \varphi^2}, \; k_c^2 > 0$$

$$\Theta(\rho,\varphi) = \left[a J_m(k_c\rho) + b Y_m(k_c\rho)\right]\left[c \sin(m\varphi) + d \cos(m\varphi)\right]$$

## ΣΧΕΣΕΙΣ ΕΓΚΑΡΣΙΩΝ ΚΑΙ ΔΙΑΜΗΚΩΝ ΠΕΔΙΩΝ

### Α) ΟΡΘΟΓΩΝΙΚΗ ΔΙΑΤΟΜΗ

$$E_x = \frac{-j}{k_C^2}\left(\beta \frac{\partial E_Z}{\partial x} + \omega\mu \frac{\partial H_Z}{\partial y}\right) \qquad E_y = \frac{-j}{k_C^2}\left(\beta \frac{\partial E_Z}{\partial y} - \omega\mu \frac{\partial H_Z}{\partial x}\right)$$

$$H_x = \frac{j}{k_C^2}\left(\omega\varepsilon \frac{\partial E_Z}{\partial y} - \beta \frac{\partial H_Z}{\partial x}\right) \qquad H_y = -\frac{j}{k_C^2}\left(\omega\varepsilon \frac{\partial E_Z}{\partial x} + \beta \frac{\partial H_Z}{\partial y}\right)$$

### Β) ΚΥΛΙΝΔΡΙΚΗ ΔΙΑΤΟΜΗ

$$E_\rho = \frac{1}{k_C^2}\left(-\gamma \frac{\partial E_Z}{\partial \rho} - \frac{j\omega\mu}{\rho}\frac{\partial H_Z}{\partial \varphi}\right) \qquad E_\varphi = \frac{1}{k_C^2}\left(-\frac{\gamma}{\rho}\frac{\partial E_Z}{\partial \varphi} + j\omega\mu \frac{\partial H_Z}{\partial \rho}\right)$$

$$H_\rho = \frac{1}{k_C^2}\left(\frac{j\omega\varepsilon}{\rho}\frac{\partial E_Z}{\partial \varphi} - \gamma \frac{\partial H_Z}{\partial \rho}\right) \qquad H_\varphi = \frac{1}{k_C^2}\left(-j\omega\varepsilon \frac{\partial E_Z}{\partial \rho} - \frac{\gamma}{\rho}\frac{\partial H_Z}{\partial \varphi}\right)$$

$$\mu_0 = 4\pi \cdot 10^{-7}, \quad \varepsilon_0 = 8{,}8542 \cdot 10^{-12}$$